## Abuse of the Equal Sign

Do you see the flaws? You can probably guess what was meant, but that's not what is written. In mathematics, we need to use precise language and notation.

Train of thought.

## - Solve $2 \mathrm{x}+1=7$ :

$$
\text { Answer: } 2 \mathrm{x}+1=7
$$

| $2 x+1=7-1$ | (not true) |
| :--- | :--- |
| $2 x=6$ |  |
| $2 x=6 / 2=3$ | (not true) |

- Find $\lim _{x \rightarrow 1} \frac{x^{2}-1}{x-1}$

$$
\text { Answer: } \quad \lim _{x \rightarrow 1} \frac{x^{2}-1}{x-1}=x+1=2 \quad \text { (neither }=\text { is true) }
$$

- Given $f(x)=x^{3}+4 x^{2}$, find $f^{\prime}(x)$.

Answer: $f^{\prime}(x)=x^{3}+4 x^{2}=3 x^{2}+8 x$ or $\begin{aligned} f(x)= & x^{3}+4 x^{2} \\ & =3 x^{2}+8 x\end{aligned}$

- Given $f(x)=x^{3}$, find $f^{\prime}(2)$.

$$
\begin{aligned}
& \text { Answer } \begin{aligned}
f^{\prime}(x) & =3 x^{2}=12 \\
f^{\prime}(2) & =3 x^{2}=12
\end{aligned} \text { (second }=\text { is not true) } \\
& \text { (neither }=\text { is true) }
\end{aligned}
$$

- Find a unit vector in the direction of $3 \vec{i}+4 \vec{j} \quad$ (this is work from a 5C student)

Answer: $3 \vec{i}+4 \vec{j}=\sqrt{3^{2}+4^{2}}=\sqrt{25}=4=\frac{3}{5} \vec{i}+\frac{4}{5} \vec{j}$

- Find the direction angle for the vector $-3 \vec{i}+3 \vec{j}$

$$
\text { Answer: } \theta=\tan ^{-1}\left(\frac{3}{-3}\right)=45^{\circ}=135^{\circ}
$$

Equals connected to nothing.

- Solve $3 x+2(x+1)=5$

$$
\text { Answer: } \quad \begin{array}{ll} 
& =3 x+2 x+2=5 \\
& =5 x+2=5 \\
& =5 x=3 \\
& =x=3 / 5
\end{array}
$$

- Given $f(x)=x^{2}$ and $g(x)=3 x+1$, find $f(g(x))$.

$$
\text { Answer: } \quad=(3 x+1)^{2}=9 x^{2}+6 x+1
$$

Nonsense in the middle of equations

- Given $f(x)=\frac{2}{x-3}$, find $f(3)$.

Answer: $f(3)=\frac{2}{3-3}=\frac{2}{0}=$ undefined $\quad(\mathrm{f}(3)$ is undefined so none of the equals make sense.)

- Find $\lim _{x \rightarrow \infty} \frac{3}{x+1}$

Answer: $\lim _{x \rightarrow \infty} \frac{3}{x+1}=\frac{3}{\infty}=0 \quad$ (Note: writing $\frac{3}{\infty}$ off to the side is helpful, it just does not belong with the equals sign)

