## Abuse of the Equal Sign

Do you see the flaws? You can probably guess what was *meant*, but that's not what is written. In mathematics, we need to use precise language and notation.

Train of thought. • Solve 2x+1=7: Answer:2x+1=7 2x+1=7-1 (not true) 2x=6 2x=6/2=3 (not true) • Find  $\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$ Answer:  $\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = x + 1 = 2 \quad \text{(neither = is true)}$ • Given  $f(x) = x^3 + 4x^2$ , find f'(x). Answer:  $f'(x) = x^3 + 4x^2 = 3x^2 + 8x$  or  $f(x) = x^3 + 4x^2$ =  $3x^2 + 8x$ • Given  $f(x) = x^3$ , find f'(2). Answer  $f'(x) = 3x^2 = 12$ (second = is not true)  $f'(2) = 3x^2 = 12$ (neither = is true) • Find a unit vector in the direction of  $3\vec{i} + 4\vec{j}$  (this is work from a 5C student) Answer:  $3\vec{i} + 4\vec{j} = \sqrt{3^2 + 4^2} = \sqrt{25} = 4 = \frac{3}{5}\vec{i} + \frac{4}{5}\vec{j}$ • Find the direction angle for the vector  $-3\vec{i} + 3\vec{j}$ Answer:  $\theta = \tan^{-1} \left( \frac{3}{-3} \right) = 45^{\circ} = 135^{\circ}$ Equals connected to nothing. • Solve 3x+2(x+1)=5Answer: = 3x+2x+2=5=5x+2=5 =5x=3=x=3/5 • Given  $f(x) = x^2$  and g(x) = 3x + 1, find f(g(x)). Answer:  $= (3x+1)^2 = 9x^2 + 6x + 1$ Nonsense in the middle of equations • Given  $f(x) = \frac{2}{x-3}$ , find f(3). Answer:  $f(3) = \frac{2}{3-3} = \frac{2}{0} = undefined$  (f(3) is undefined so none of the equals make sense.) • Find  $\lim_{x \to \infty} \frac{3}{x+1}$ Answer:  $\lim_{x \to \infty} \frac{3}{x+1} = \frac{3}{\infty} = 0$  (Note: writing  $\frac{3}{\infty}$  off to the side is helpful, it just does not belong with the equals sign)